

#

### Semester One Examination, 2019

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNIT 3**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

 In words

 Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Workingtime (minutes) | Marks available | Percentage of examination |
| Section One:Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (5 marks)

Fuel flows into a storage tank that is initially empty at a rate of litres per minute, where is the time in minutes and .

(a) Determine how much fuel is in the tank after minutes. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral evaluates integral |

(b) If the tank is completely full after minutes, determine the time required for the tank to become one-fifth full. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates total volume writes integral and equates to one-fifth volume evaluates time |

Question 10 (7 marks)

 is a uniform discrete random variable where .

(a) Determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  correct value |

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  correct probability |

(b) Calculate the exact value of

(i) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression  |

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ standard deviation  |

Question 11 (8 marks)

The potential difference, volts, across the terminals of an electrical capacitor seconds after it begins to discharge through a resistor can be modelled by the equation

 is the initial potential difference and is a constant that depends on the size of the capacitor and the resistor.

(a) If volts and , determine

(i) the potential difference across the capacitor minutes after discharge began.

 (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses correct time calculates correct voltage |

(ii) the time taken for the potential difference to drop from to volts. (3 marks)

|  |
| --- |
| **Solution** |
| When and when .Hence takes  |
| **Specific behaviours** |
| ✓ calculates first time calculates second time calculates difference, correct to at least 1 dp |

(iii) the rate of change of when the potential difference is volts. (1 mark)

|  |
| --- |
| **Solution** |
|   |
| **Specific behaviours** |
| ✓ calculates rate |

(b) Another capacitor takes seconds for its maximum potential difference to halve. It is instantly recharged to its maximum every minutes, which is the time required for the potential difference to fall from its maximum to volts. Determine the maximum potential difference for this capacitor. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines  determines  |

Question 12 (8 marks)

A manufacturing process begins and the rate at which it produces gas after minutes () is modelled by

(a) State the maximum rate that gas can be produced at. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct rate |

(b) Calculate the rate that gas is being produced after minutes. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct rate (exact or at least 1dp) |

(c) Use the increments formula to determine the approximate change in between and seconds after production began. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct  correct  correct change |

(d) Use the increments formula to determine the approximate volume of gas produced in the seconds following . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct use of increments formula uses correct and  correct estimate (at least 2dp) |

Question 13 (5 marks)

The function is shown below.



(a) Use the sum of the areas of the inscribed rectangles shown in the diagram to explain why . (2 marks)

|  |
| --- |
| **Solution** |
| Since the definite integral represents area under curve, the value of the integral must be more than . |
| **Specific behaviours** |
|  shows calculation for area underestimate explains area under curve must be more than underestimate |

(b) Use the average of the sum of the areas of the inscribed rectangles and the sum of the areas of the circumscribed rectangles shown to determine an estimate for .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows calculation for underestimate calculates average |

 (2 marks)

(c) Suggest a modification to the method used in (b) to achieve a better estimate for . (1 mark)

|  |
| --- |
| **Solution** |
| Use a larger number of narrower rectangles. |
| **Specific behaviours** |
| ✓ sensible modification |

Question 14 (7 marks)

Let .

(a) Sketch the graph of on the axes below. (2 marks)



|  |
| --- |
| **Solution (a)** |
| See graph |
| **Specific behaviours** |
| ✓ smooth curve, asymptotic for  passes through  |

|  |
| --- |
| **Solution (b)** |
| See graph |
| **Specific behaviours** |
| ✓ straight line, correct intercepts clearly shades required area |

(b) The line is tangential to the curve at , and it intersects the -axis at the point . Add the line to the graph above and shade the area enclosed by the line, the curve and . (2 marks)

(c) Determine the area enclosed by the line, the curve and . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates value of  writes integral using difference of functions evaluates integral |

Question 15 (7 marks)

The graph of is shown below for .



The area trapped between the -axis and the curve for regions and are and square units respectively.

(a) Determine the value of

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows sum of signed areas correct value |

(iii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ area of rectangle correct value |

(iv) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows second integral is zero correct value |

Question 16 (12 marks)

The random variable is the number of goals scored by a team in a soccer match, where

(a) Determine the probability that the team scores at least one goal in a match. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  correct probability |

The random variable is the bonus each player is paid after a match, depending on the number of goals the team scored. For four or more goals is paid, for two or three goals is paid and for one goal is paid. No bonus is paid if no goals are scored.

(b) Complete the probability distribution table for . (3 marks)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Goals scored |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ missing values and   |

(c) Calculate

(i) the mean bonus paid per match. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression mean |

(ii) the standard deviation of the bonus paid per match. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ variance standard deviation |

(d) The owner of the team plans to increase the current bonuses by next season (so that the players will get a bonus of even when no goals are scored) and then further raise them by the following season. Determine the mean and standard deviation of the bonus paid per match after both changes are implemented. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct multiplier new mean new standard deviation |

Question 17 (9 marks)

Seeds were planted in rows of five and the number of seeds that germinated in each of the rows are summarised below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of germinating seeds |  |  |  |  |  |  |
| Number of rows |  |  |  |  |  |  |

(a) Use the results in the table to determine

(i) the probability that no more than seeds germinated in a randomly selected row.

 (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(ii) the mean number of seeds that germinated per row. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct mean |

(b) Another row of five seeds is planted. Determine the probability that no more than seeds germinate in this row if the number that germinate per row is binomially distributed with the above mean. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates  correct probability |

Suppose it is known that of all seeds planted will germinate and that seeds are now planted in rows of .

(c) Assuming that seeds germinate independently of each other, determine

(i) the most likely number of seeds to germinate in a row. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(ii) the probability that at least seeds germinate in a randomly chosen row.

 (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states distribution correct probability |

(iii) the probability that in eight randomly chosen rows, exactly six rows have at least seeds germinating in them. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states distribution correct probability |

Question 18 (9 marks)

The graph of is shown below.



Let be defined by the integral for .

(a) Use the graph of to identify all the turning points of the graph of , stating the -coordinate and nature of each point. (2 marks)

|  |
| --- |
| **Solution** |
| At there is a maximumAt there is a minimum |
| **Specific behaviours** |
| ✓ location of maximum location of minimum |

It is also known that the .

(b) Using the graph of or otherwise, explain why . (2 marks)

|  |
| --- |
| **Solution** |
| . From the graph and hence . |
| **Specific behaviours** |
| ✓ shows use of and integral explanation, using area and  |

(c) Sketch the graph of on the axes below, indicating and labelling the location of all key features. (5 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ Labelled point of inflection at origin Labelled roots, as indicated Curve with labelled maximum Curve with labelled minimum Straight line, as indicated |

Question 19 (7 marks)

A small body has displacement when and moves along the -axis so that its velocity after seconds is given by

(a) Determine an equation for , the displacement of the body after seconds. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ integrates correctly attempts to find constant using substitution correct equation |

(b) Describe, with justification, how the speed of the body is changing when . (4 marks)

|  |
| --- |
| **Solution** |
| Since the body has a negative velocity but a positive acceleration then its speed is decreasing when . |
| **Specific behaviours** |
| ✓ clearly shows is negative expression for  clearly shows is positive explains decreasing speed using signs of and  |

Question 20 (7 marks)

An aquarium, with a volume of cm3, takes the shape of a rectangular prism with square ends of side cm and no top. The glass for the base costs cents per square cm and for the four vertical sides costs cents per square cm. The cost of glue to join the edges of two adjacent pieces of glass is cents per cm. Assume the glass has negligible thickness and ignore any other costs.

(a) Show that , where is the cost, in dollars, to make the aquarium. (4 marks)

|  |
| --- |
| **Solution** |
| Let be third length, so that Cost of glass: Cost of edges: Total cost:  |
| **Specific behaviours** |
| ✓ expression for third side in terms of  cost of glass (simplification not required) cost of edges (simplification not required) sums and converts to dollars |

(b) Show use of a calculus method to determine the minimum cost of making the aquarium.

 (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows marginal cost determines value of so that marginal cost is zero determines minimum cost, to nearest cent. |

Question 21 (7 marks)

(a) Given that and , determine the exact value of

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ recognises  correct value |

(b) Given that , and , determine . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows  uses to obtain with constant  integrates again to obtain  evaluates constant and writes expression for  |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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